$$q = \frac{K_\mu (I_{A'} - I_{A'''}) + (I_{B'} - I_{B'''})}{x_{B''} - x_{B'}}$$

or

$$q = \frac{1 + K_\mu}{x_{B''} - x_{B'}} (I_{C'} - I_C).$$

Thus, the given procedure can be used in the calculation of installations for drying gas suspensions and the determination of the consumption of heat, air, and dust in drying.

NOTATION

c, mass specific heat, kJ/kg·°C; G, mass flow rate, kg/sec; I, enthalpy of a unit mass, kJ/kg; Q, heat, kW; t, temperature, °C; W, mass flow rate of moisture, kg/sec; x, moisture content per unit mass of dry material, kg/kg; μ, specific, mass, flow-rate concentration of dust, kg/kg dry air; φ, relative humidity, %; r, heat of vaporization of water, kJ/kg. Indices: v, vapor; a, air; s, dust; as, air substituting for dust; no prime, at entrance to air chamber; '", at exit from air chamber; ', at entrance to drying chamber; ", at exit from drying chamber.

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OPTIMUM CONTROL OF MULTIZONE CONVECTIVE DRYERS

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The problem of the optimum control of multizone convective dryers is solved. A dryer used for dying pipes is considered as an example.

Multizone convective dryers are widely used for drying materials in different branches of industry: chemical, construction materials, etc. In these dryers the drying agent is supplied by zones, and to each zone one can supply "fresh" drying agent, that "exhausted" for all the other zones, and "cold" air. Transfer of the material is accomplished with transporting devices.

As the optimality criterion characterizing the energy expenditure on drying we take a criterion allowing for the consumption of "fresh" drying agent and the consumption connected with the flows of drying agent from one zone over to another:

$$I = \sum_{j=1}^{n} \left[c_{1j} G_{gj} + \sum_{\substack{i=1 \\ i \neq j}}^{n} (c_{2ji} G_{gji} + c_{3ji} R_{ji}) \right], \tag{1}$$

where $R_{ji} = 0$ at $G_{gji} = 0$ and $R_{ji} = 1$ at $G_{gji} > 0$. The first term $c_{1j}G_{gj}$ in Eq. (1) characterizes the expenditure of "fresh" drying agent, with the coefficient c_{1j} depending on the potentialities of the heated gas as a drying agent; in particular, the value of c_{1j} increases with an increase in the gas temperature. The other two terms in Eq. (1), $c_{2ji}G_{gji}$ and $c_{3ji}R_{ji}$, characterize the expenditures connected with the flows of drying agent from the i-th to the j-th zone, with the value of the coefficient c_{3ji} being determined by the capital expenditures on the installation of the corresponding blower and the expenditures on the idle

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time of its electric motor, while the value of the coefficient c_{2ji} depends on the hydrodynamic conditions of the flow between zones.

The controlling actions are the flow rates of "fresh" drying agent into each zone of the dryer, the flow rates of "exhausted" drying agent transferred between zones, and, in the design of dryers, also the sizes of the zones. Limits, which are determined primarily by the mathematical model of the dryer, are imposed on the parameters characterizing the drying process, including the controlling actions. In constructing a mathematical model we make the following basic assumptions:

1) the dryer consists of three interconnected heat- and mass-accumulating capacities: the material, the drying agent, and the transporting device;
2) the parameters of the material (moisture content and temperature) and the transporting devices (temperature) are distributed over the coordinate along the motion of the material and the transporting devices and are concentrated in the transverse direction;
3) the parameters of the drying agent (temperature and moisture content) are concentrated by zones;
4) the heat capacity of the material is a linear function of its temperature;
5) the kinetics of the drying process is described by the following equation:

$$\partial w/\partial \tau = kw. \tag{2}$$

With allowance for the adopted assumptions, to obtain the mathematical model of a multizone convective dryer we must use, as indicated in [1], the equations of material balance (with respect to moisture) for the material and the drying agent and of heat balance for the material, the drying agent, and the transporting devices for each j-th zone:

$$f_1 = \frac{\partial w}{\partial x} = a_1 k_j w, \tag{3}$$

$$f_2 = \frac{\partial t_m}{\partial x} = \frac{a_{2j} t_m + a_3 k_j w + a_{4j} t_{1gj} + a_{4j} t_{2gj}}{a_5 + a_6 t_m + a_7 w}, \tag{4}$$

$$f_3 = \frac{\partial t_t}{\partial x} = a_{8j} t_t + a_{9j} t_{1gj} + a_{9j} t_{2gj}, \tag{5}$$

$$\sum_{\substack{i=1\\ i\neq j}}^{n} G_{gji}(0.24\, t_{2gi} + 0.47\, t_{2gi} d_{2i} + 595\, d_{2i}) + a_{10} G_{gj} + a_{16} G_{aj} - G_{gj,\Sigma}(0.24\, t_{2gj} + 0.47\, t_{2gj} d_{2j} + 595\, d_{2j})$$
$$+ a_{11j} t_{1gj} + a_{11j} t_{2gj} - a_{2j} t_{mj,av} + a_{17j} t_{tj,av} + a_{12} k_j w_{j,av} + a_{13} k_j w_{j,av} t_{mj,av} + a_{14j} = 0, \tag{6}$$

$$\sum_{\substack{i=1\\ i\neq j}}^{n} G_{gji} d_{2i} + a_{21j} G_{gj} + a_{18} G_{aj} - G_{gj,\Sigma} d_{2j} + a_{15} k_j w_{j,av} = 0, \tag{7}$$

$$\text{at } x = 0 \quad w = w_1,\ t_m = t_{1m},\ t_t = t_{1t}. \tag{8}$$

Here

$$G_{gj,\Sigma} = \sum_{\substack{i=1\\ i\neq j}}^{n} G_{gji} + a_{19j} G_{gj} + G_{aj}$$

$$t_{1gj} = \frac{0.24 \sum_{\substack{i=1\\ i\neq j}}^{n} G_{gji} t_{2gi} + a_{10} G_{gj} + 0.24\, a_{20} G_{gj}}{0.24\, G_{gj,\Sigma}};$$

$$w_{j,av} = \int_{x_{j-1}}^{x_j} w\,dx \Big/ (x_j - x_{j-1});\quad t_{mj,av} = \int_{x_{j-1}}^{x_j} t_m\,dx \Big/ (x_j - x_{j-1});$$

$$t_{tj,av} = \int\limits_{x_{j-1}}^{x_j} t_t dx \Big/ (x_j - x_{j-1}).$$

The following limits are imposed on the parameters characterizing the drying process in multizone convective dryers:

1) the moisture content of the material at the exit from the dryer must not exceed some value w_f determined by technological regulations:

$$\text{at } x = 1 \quad w \leqslant w_f; \tag{9}$$

2) the rate of change of the moisture content of the material along the length of the dryer must not exceed the drying rate N_{cr} determined on the basis of the equation for the limiting safe drying curve:

$$\frac{\partial w}{\partial x} \leqslant N_{cr}; \tag{10}$$

3) the total flow rate of drying agent from the j-th zone into all the other i-th zones obviously cannot exceed the flow rate of drying agent entering this j-th zone, i.e., we have the inequality

$$\sum_{\substack{i=1 \\ i \neq j}}^{n} G_{gij} \leqslant G_{gj,\Sigma}. \tag{11}$$

So, the problem of the optimum control of multizone convective dryers can be formulated as follows: to provide the minimum energy expenditure on drying in accordance with the optimality criterion (1) in the presence of the connections (3)-(7) and the limits (9)-(11). The controlling actions are G_{gj} and G_{gji}, as well as x_j in the construction of dryers.

The difficulties in the solution of the stated problem are connected with the presence of limits of different types and with the presence of both continuous and discrete functions in these limits. As indicated in [1, 2], to solve this problem one must use a method based on the use of generalized functions and the method of Lagrangian multipliers. To allow for the limits (10) and (11) we use the method of penalty functions. Then the functional (1) takes the form

$$I = \sum_{j=1}^{n}\Big[c_{1j}G_{gj} + \sum_{\substack{i=1 \\ i\neq j}}^{n}(c_{2ji}G_{gji} + c_{3ji}R_{ji})\Big] + I_0 + \sum_{j=1}^{n} c_{5j}\Big[\Big(G_{gj,\Sigma} - \sum_{\substack{i=1 \\ i\neq j}}^{n} G_{gij}\Big) - \Big|G_{gj,\Sigma} - \sum_{\substack{i=1 \\ i\neq j}}^{n} G_{gij}\Big|\Big]^2, \tag{12}$$

where

$$I_0 = c_4[(N_{cr} - a_1 k_j w) - |N_{cr} - a_1 k_j w|]^2.$$

The values of the coefficients c_4 and c_{5j} are chosen as arbitrary but sufficiently large [2]. We assume that Eqs. (6) and (7) are satisfied through the choice of the corresponding values of t_{2gj} and d_{2j}, for which these equations must be solved for t_{2gj} and d_{2j} by numerical methods. The expressions for the Lagrangian functional and the auxiliary variables ψ_1, ψ_2, and ψ_3 have the form

$$S = \sum_{j=1}^{n}\Big[\int\limits_{x_{j-1}}^{x_j}\Big(\frac{\partial\psi_1}{\partial x} w + \psi_1 f_1 + \frac{\partial\psi_2}{\partial x} t_m + \psi_2 f_2 + \frac{\partial\psi_3}{\partial x} t_t + \psi_3 f_3\Big)dx\Big] - I, \tag{13}$$

$$\frac{\partial\psi_1}{\partial x} + \psi_1\frac{\partial f_1}{\partial w} + \psi_2\frac{\partial f_2}{\partial w} + \frac{\partial I_0}{\partial w} = 0, \tag{14}$$

$$\frac{\partial\psi_2}{\partial x} + \psi_2\frac{\partial f_2}{\partial t_m} = 0, \tag{15}$$

$$\frac{\partial\psi_3}{\partial x} + \psi_3\frac{\partial f_3}{\partial t_t} = 0. \tag{16}$$

The boundary conditions for the auxiliary variables ψ_1, ψ_2, and ψ_3 at the right end of the trajectory (at x = 1) are determined on the basis of the boundary conditions (at x = 1) for w, t_m, and l_t. For w this condition is determined by Eq. (9), while strict limits are not imposed on the values of the temperatures of the material and the transporting device at the exit from the dryer (t_m and t_t at x = 1). As a result, we have

$$\psi_1(1)=0 \quad \text{at } w \leqslant w_f,$$
$$\psi_1(1)=-b_0 \quad \text{at } w > w_f, \quad \text{where } b_0^{(l+1)} = b_0^l + b_1[w^l(1)-w_f], \tag{17}$$
$$\psi_2(1)=0,$$
$$\psi_3(1)=0.$$

Here l is the iteration number in the numerical solution of the problem, and b_0 and b_1 are coefficients whose values are chosen so as to assure the rapid convergence of the iteration procedure.

Solving Eqs. (15) and (16) with the condition (17), we obtain $\psi_2 = 0$ and $\psi_3 = 0$. With allowance for the fact that $\psi_2 = 0$ and $\psi_3 = 0$, Eq. (14) with the condition (17) is solved in quadratures, but the resulting integrals are not taken and can only be determined by numerical methods.

We use the gradient method to minimize the functional. The numerical procedure for solving the optimization problem consists in the following.

1. We assign the initial values G^o_{gj}, G^o_{gji}, and x^o_j. By solving the system of equations (3)-(7) with the initial conditions (8) for all the j-th zones of the dryer, we determine the values $w^o(x)$, $t^o_m(x)$, $t^o_g(x)$, t^o_{2gj}, and d^o_{2j}. Equations (6) and (7) are solved by Newton's method while Eqs. (3)-(5) are solved by the Runge–Kutta method. Iteration procedures are used in this case, since for each j-th zone in the solution of Eqs. (6) and (7) both the values $w_{j,av}$, $t_{mj,av}$, and $t_{tj,av}$ and the values t_{2gi} and d_{2i} are known for those i-th zones for which $i > j$.

2. We calculate $\psi^o_1(x)$ by solving Eq. (14) with the condition (17) and the values $G_{gj} = G^o_{gj}$, $G_{gji} = G^o_{gij}$, $x_j = x^o_j$, $w(x) = w^o(x)$, $t_m(x) = t^o_m(x)$, $t_t(x) = t^o_t(x)$, $t_{2gj} = t^o_{2gj}$, and $d_{2j} = d^o_{2j}$.

3. We determine the values of the derivatives

$$\frac{\partial S}{\partial G_{gj}}(G^0_{gj},\ G^0_{gji},\ x^0_j,\ w^0,\ t^0_m,\ t^0_t,\ t^0_{2gj},\ d^0_{2j},\ \psi^0_1) = \delta_{G_j},$$
$$\frac{\partial S}{\partial G_{gji}}(G^0_{gj},\ G^0_{gji},\ x^0_j,\ w^0,\ t^0_m,\ t^0_t,\ t^0_{2gj},\ d^0_{2j},\ \psi^0_1) = \delta_{G_{ji}},$$
$$\frac{\partial S}{\partial x_j}(G^0_{gj},\ G^0_{gji},\ x^0_j,\ w^0,\ t^0_m,\ t^0_t,\ t^0_{2gj},\ d^0_{2j},\ \psi^0_1) = \delta_{x_j}.$$

4. We determine the new values $G^1_{gj} = G^o_{gj} + \varepsilon_{Gj}\delta_{Gj}$, $G^1_{gji} = G^o_{gji} + \varepsilon_{Gji}\delta_{Gji}$, and $x^1_j = x^o_j + \varepsilon_{xj}\delta_{xj}$. The values of the coefficients ε_{Gj}, ε_{Gji}, and ε_{xj} are chosen from the condition of convergence of the search procedure.

The difference between the values of the functional (1) in two successive steps can be taken as the criterion for ending the search procedure. The search ends when this difference does not exceed some value Δ chosen in advance.

The procedure given above for solving the optimization problem was used to calculate the optimum operating regimes for a multizone convective dryer used to dry pipes in the construction material industry.

The following are the main characteristics of the dryer: number of zones $n = 6$; flow rate of material 9050 m/h; amount of material inside the dryer 262,000 kg; moisture content of material at entrance to dryer $w_1 = 22.1\%$; coordinates of boundaries of zones: $x = 0.14$, $x = 0.28$, $x = 0.465$, $x = 0.65$, $x = 0.84$, $x = 1$; flow rates of "fresh" drying agent by zones: $G_{g1} = 0$, $G_{g2} = 7000$ kg/h, $G_{g3} = 7290$ kg/h, $G_{g4} = 0$, $G_{g5} = 15{,}340$ kg/h, $G_{g6} = 12{,}370$ kg/h; flow rates of "exhausted" drying agent flowing between zones: $G_{g46} = 12{,}370$ kg/h, $G_{g45} = 15{,}340$ kg/h, $G_{g34} = 27{,}710$ kg/h, $G_{g23} = 35{,}000$ kg/h, $G_{g12} = 42{,}000$ kg/h, other $G_{gji} = 0$; no cold air is supplied to any dryer zones.

For the indicated construction materials the equations for determining the drying coefficient k and the safe limiting drying curve have the following form [3]:

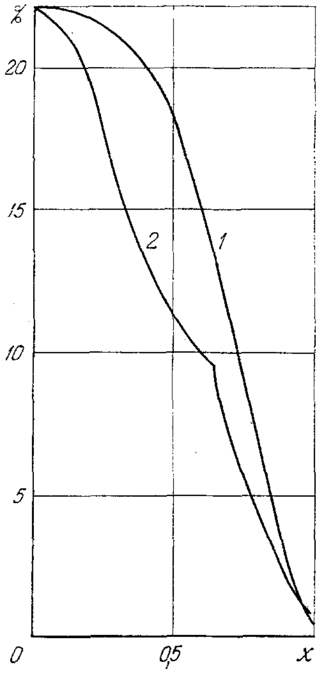

Fig. 1. Curves of variation of moisture content of the material along the length of the dryer: 1) optimum; 2) actual regime.

$$\text{at } w \geqslant a_{22}w_1: \quad k = \bar{k} = \frac{B}{R_v} v^{a_{23}} \Delta t^{a_{24}}, \tag{18}$$

$$\text{at } a_{25}w_1 \leqslant w < a_{22}w_1 \text{ and } \Delta t \leqslant a_{26}: \; k = a_{27}\bar{k},$$

$$\text{at } a_{25}w_1 \leqslant w < a_{22}w_1 \text{ and } \Delta t > a_{26}:$$

$$k = a_{28}\bar{k}\Delta t^{a_{29}},$$

$$\text{at } w < a_{25}w_1: \quad k = a_{30}\bar{k}\Delta t^{a_{29}}; \tag{19}$$

$$\text{at } w \geqslant w_{\text{cr}}: \quad N_{\text{cr}} = a_{31},$$

$$\text{at } w < w_{\text{cr}}: \quad N_{\text{cr}} = a_{32} \exp(a_{33}w).$$

Here

$$\Delta t = t_{\text{g}} - t_{\text{wet}}; \; t_{\text{g}} = \frac{t_{1\text{g}} + t_{2\text{g}}}{2};$$

$$v = G_{\text{g},\Sigma}/4660\, F \frac{273}{273 + t_{\text{g}}}.$$

In calculating the optimum regimes of the dryers indicated above we also assume that the heat losses to the ambient medium and to heating the material and the transporting devices are constant, the boundaries x_j of the zones are fixed, and the coefficients c_{3ji} in Eq. (1) for the functional equal zero. The calculations were made on BÉSM-6 and M-4030 computers by a program compiled in the FORTRAN algorithmic language.

The drying curve corresponding to the above-indicated technical characteristics of the dryer and the drying curve corresponding to the optimum regime for the drying process calculated by the algorithm described above are presented in Fig. 1.

In the optimum drying regime the controlling actions have the following values: flow rates of "fresh" drying agent by zones: $G_{g1} = 0$; $G_{g2} = 1000$ kg/h; $G_{g3} = 1500$ kg/h; $G_{g4} = 4000$ kg/h; $G_{g5} = 13{,}000$ kg/h; $G_{g6} = 20{,}500$ kg/h; flow rate of "exhausted" drying agent flowing between zones: $G_{g56} = 20{,}500$ kg/h; $G_{g45} = 33{,}500$ kg/h; $G_{g34} = 37{,}500$ kg/h; $G_{g23} = 39{,}000$ kg/h; $G_{g12} = 40{,}000$ kg/h; other $G_{gji} = 0$.

Thus, in the optimum drying mode the total flow rate of "fresh" drying agent was reduced by 2000 kg/h, i.e., by about 5%, while the value of the functional I with $c_{1j} = 1$ and $c_{2ji} = 0.005$ was reduced by about 4.5%.

NOTATION

G_{gj}, flow rate of "fresh" drying agent into j-th zone; G_{gji}, flow rate of "exhausted" drying agent of i-th zone into j-th zone; c_{1j}, c_{2ji}, c_{3ji}, constants used to "weight" different kinds of expenditures; w, moisture content of material; k, drying coefficient; τ, time; t_{wet}, t_m, t_{1m}, t_t, t_{1t}, t_{2g}, temperatures of wet-bulb thermometer, of material along length of and at entrance to dryer, of transporting devices along length of and at entrance to dryer, and of "exhausted" drying agent; G_a, flow rate of cold air; d_2, moisture content of "exhausted" drying agent; n, number of zones in dryer; x, x_{j-1}, x_j, coordinates of current, initial, and end points of j-th zone of dryer; α_1–α_{32}, constants; B, coefficient depending on mass composition of material; R_U, hydraulic radius; Δt, drying potential; v, velocity of drying agent; F, cross-sectional area of dryer; w_{cr}, critical moisture content of material.

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WAVE REGIME OF FILTRATION OF SUSPENSIONS

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It is shown that a wave regime can exist for models of filtration of one-component and binary suspensions which take into account the formation of a deposit in stagnant and flowing regions of the porous bed. Theoretical formulas for the concentration in the solution and deposit phases are obtained.

1. Filtration of suspensions in a porous medium in order to purify solutions or extract components of the disperse phase is a common mass-transfer process. If clarification is effected by adhesion throughout the porous bed without formation of a film on the filter surface and at a constant filtration rate, engineering methods of calculating the time of protective action t_{pr} of the filter and the head (H) loss time are based on two empirical relations [1]:

$$t_{pr} = kl - \tau, \tag{1}$$

$$t_h = \frac{H - h_0}{m}, \tag{2}$$

which are applicable for sufficiently long beds and sufficiently long filtering cycles. A method of experimental determination of the constants contained in Eqs. (1) and (2) has been developed by technological simulation of the actual process [1].

Equation (1) was first proposed by Shilov [2] for calculation of sorption filters. It was subsequently shown in sorption theory that this equation is valid for a wave regime when a concentration front moving through the bed at constant velocity $n = k^{-1}$ is formed, and that a sufficient condition for occurrence of the wave regime is convexity of the sorption isotherm [3]. In filtration dynamics, however, the question of the existence of a wave regime has not been investigated. In addition, models [4, 5] which generally have no wave regime are widely used.

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